A **perfect number** is one such that all its factors, including one but excluding itself, add up to itself. For example, 6 is a perfect number since 6 has factors 1, 2, 3 and 6 and $1 + 2 + 3 = 6$. In fact, 6 is the smallest perfect number.

**Euclid** proved in his *Elements* that a number of the form $2^{p-1}(2^p - 1)$ is a perfect number if $2^p - 1$ is a prime number.

I used this fact to write a programme in Basic to find perfect numbers but first we need a programme on prime numbers for checking if $2^p - 1$ is prime.

```basic
ready.
10 rem prime numbers
11 rem to calculate prime numbers up to a
20 input a
22 if a=2 then print"2": goto 100
25 print"2 3";
30 for i=2 to a
40 if i=a then 100
50 for d=2 to int(sqr(i)+2)
60 if i/d=int(i/d) then 90
70 next d
80 printi;
90 next i
100 end
```

So now let us see how we can use lines 40-60 to find perfect numbers.

```basic
10 rem perfect numbers
15 rem to calculate perfect numbers
20 input n
30 if n<6 then print "none": goto 200
35 if n=6 then print "6 only": goto 200
40 print"6";
45 for i=3 to 26
```

---

46 rem limit n to $2^{25} \times (2^{26} - 1)$
47 let y=$2^i - 1$
50 rem next loop is to check if $2^i-1$ is prime
52 for l=2 to int(sqr(y))
53 if y/l=int(y/l) then 70
54 if y*2^i-1>n then 200
55 next l
57 print",";y*2^i-1;
70 next i
200 print
201 print"(this program was written on 26/8/83)"
300 end

Unfortunately, line 45 limits us to $2^{25} \times (2^{26} - 1)$, but then the computer has a limited range of numbers: it will never get to $2^{25} \times (2^{26} - 1)$ anyway. I have computed perfect numbers up to $10^{13}$.

6, 28, 496, 8128, 33 550 336, 8.58986906e + 09, 1.37438691e + 11

The last two, of course, are only approximations to the actual perfect numbers and are unacceptable in this form. 8.58986906e + 09 = 8 589 869 060 when the last two figures are in doubt. In fact it is 8 589 869 056.

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